

# THE SCATTERING APPROACH TO THE CASIMIR FORCE

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We present the scattering approach which is nowadays the best tool for describing the Casimir force in realistic experimental configurations. After reminders on the simple geometries of 1d space and specular scatterers in 3d space, we discuss the case of stationary arbitrarily shaped mirrors in electromagnetic vacuum. We then review specific calculations based on the scattering approach, dealing for example with the forces or torques between nanostructured surfaces and with the force between a plane and a sphere. In these various cases, we account for the material dependence of the forces, and show that the geometry dependence goes beyond the trivial *Proximity Force Approximation* often used for discussing experiments.

## The many facets of the Casimir effect

The Casimir effect [1] is a jewel with many facets. First, it is an observable effect of vacuum fluctuations in the mesoscopic world, which deserves careful attention as a crucial prediction of quantum field theory [2–7].

Then, it is also a fascinating interface between quantum field theory and other important aspects of fundamental physics. It has connections with the puzzles of gravitational physics through the problem of vacuum energy [8, 9] as well as with the principle of relativity of motion through the dynamical Casimir-like effects [10–12]. Effects beyond the Proximity Force Approximation also make apparent the extremely rich interplay of vacuum energy with geometry (references and more discussions below).

Casimir physics also plays an important role in the tests of gravity at sub-millimeter ranges [13, 14]. Strong constraints have been obtained in short range Cavendish-like experiments [15] : Should an hypothetical new force have a Yukawa-like form, its strength could not be larger than that of gravity if the range is larger than  $56\mu\text{m}$ . For scales of the order of the micrometer, similar tests are performed by comparing with theory the results of Casimir force measurements [16, 17]. At even shorter scales, the same can be done with atomic [18] or nuclear [19] force measurements.

Finally, the Casimir force and closely related Van der Waals force are dominant at micron or sub-micron distances, which entails that they have strong connections with various important domains, such as atomic and molecular physics, condensed matter and surface physics, chemical and biological physics, micro- and nano-technology [20].

## Comparison of the Casimir force measurements with theory

In short-range gravity tests, the new force would appear as a difference between the experimental result  $F_{\text{exp}}$  and theoretical prediction  $F_{\text{th}}$ . This implies that  $F_{\text{th}}$  and

$F_{\text{exp}}$  have to be assessed independently from each other and should forbid anyone to use theory-experiment comparison for proving (or disproving) some specific experimental result or theoretical model.

Casimir calculated the force between a pair of perfectly smooth, flat and parallel plates in the limit of zero temperature and perfect reflection. He found universal expressions for the force  $F_{\text{Cas}}$  and energy  $E_{\text{Cas}}$

$$F_{\text{Cas}} = \frac{\hbar c \pi^2 A}{240 L^4} \quad , \quad E_{\text{Cas}} = -\frac{\hbar c \pi^2 A}{720 L^3} \quad (1)$$

with  $L$  the distance,  $A$  the area,  $c$  the speed of light and  $\hbar$  the Planck constant. This universality is explained by the saturation of the optical response of perfect mirrors which reflect 100% (no less, no more) of the incoming fields. Clearly, this idealization does not correspond to any real mirror. In fact, the effect of imperfect reflection is large in most experiments, and a precise knowledge of its frequency dependence is essential for obtaining a reliable theoretical prediction for the Casimir force [21].

The most precise experiments are performed with metallic mirrors which are good reflectors only at frequencies smaller than their plasma frequency  $\omega_P$ . Their optical response is described by a reduced dielectric function usually written at imaginary frequencies  $\omega = i\xi$  as

$$\varepsilon[i\xi] = \hat{\varepsilon}[i\xi] + \frac{\sigma[i\xi]}{\xi} \quad , \quad \sigma[i\xi] = \frac{\omega_P^2}{\xi + \gamma} \quad (2)$$

The function  $\hat{\varepsilon}[i\xi]$  represents the contribution of inter-band transitions and it is regular at the limit  $\xi \rightarrow 0$ . Meanwhile  $\sigma[i\xi]$  is the reduced conductivity ( $\sigma$  is measured as a frequency and the SI conductivity is  $\epsilon_0\sigma$ ) which describes the contribution of the conduction electrons.

A simplified description corresponds to the lossless limit  $\gamma \rightarrow 0$  often called the plasma model. As  $\gamma$  is much smaller than  $\omega_P$  for a metal such as Gold, this simple model captures the main effect of imperfect reflection. However it cannot be considered as an accurate description since a much better fit of tabulated optical data is obtained with a non null value of  $\gamma$  [21]. Furthermore, the Drude model meets the important property of ordinary metals which have a finite static conductivity  $\sigma_0 = \frac{\omega_P^2}{\gamma}$ ,

in contrast to the lossless limit which corresponds to an infinite value for  $\sigma_0$ .

Another correction to the Casimir expressions is associated with the effect of thermal fluctuations [22, 23] which is correlated to the effect of imperfect reflection [24]. Bostrom and Sernelius have remarked that the small non zero value of  $\gamma$  had a significant effect on the force evaluation at  $T \neq 0$  [25]. This remark has led to a blossoming of contradictory papers (see references in [26–28]). The current status of Casimir experiments appears to favor predictions obtained with  $\gamma = 0$  rather than those corresponding to the expected  $\gamma \neq 0$  (see Fig.1 in [29]). Note that the ratio between the prediction at  $\gamma = 0$  with that at  $\gamma \neq 0$  reaches a factor 2 at the limit of large temperatures or large distances, although it is not possible to test this striking prediction with current experiments which do not explore this domain.

At this point, it is worth emphasizing that microscopic descriptions of the Casimir interaction between two metallic bulks lead to predictions agreeing with the lossy Drude model rather than the lossless plasma model at the limit of large temperatures or large distances [30–32]. At the end of this discussion, we thus have to face a worrying situation with a lasting discrepancy between theory and experiment. This discrepancy may have various origins, in particular artefacts in the experiments or inaccuracies in the calculations. A more subtle but maybe more probable possibility is that there exist yet unmastered differences between the situations studied in theory and the experimental realizations.

### The role of geometry

The geometry of Casimir experiments might play an important role in this context. Precise experiments are indeed performed between a plane and a sphere whereas calculations are often devoted to the geometry of two parallel planes. The estimation of the force in the plane-sphere geometry involves the so-called *Proximity Force Approximation* (PFA) [33] which amounts to averaging over the distribution of local inter-plate distances the force calculated in the two-planes geometry, the latter being deduced from the Lifshitz formula [34, 35].

This trivial treatment of geometry cannot reproduce the rich interconnection expected to take place between the Casimir effect and geometry [36]. In the plane-sphere geometry in particular, the PFA can only be valid when the radius  $R$  is much larger than the separation  $L$  [37]. But even if this limit is met in experiments, the PFA does not tell one what is its accuracy for a given value of  $L/R$  or whether this accuracy depends on the material properties of the mirror. Answers to these questions can only be obtained by pushing the theory beyond the PFA, which has been done in the past few years (see references in [38–42]). In fact, it is only very recently that these calculations have been done with plane and spherical metallic plates coupled to electromagnetic vacuum

[43], thus opening the way to a comparison with experimental studies of PFA in the plane-sphere geometry [44].

Another specific geometry of great interest is that of surfaces with periodic corrugations. As lateral translation symmetry is broken, the Casimir force contains a lateral component which is smaller than the normal one, but has nevertheless been measured in dedicated experiments [45]. Calculations beyond the PFA have first been performed with the simplifying assumptions of perfect reflection [46] or shallow corrugations [47–49]. As expected, the PFA was found to be accurate only at the limit of large corrugation wavelengths. Very recently, experiments have been able to probe the beyond-PFA regime [50, 51] and it also became possible to calculate the forces between real mirrors with deep corrugations [52]. More discussions on these topics will be presented below.

### Introduction to the scattering approach

The best tool available for addressing these questions is the scattering approach. We begin the review of this approach by an introduction considering the two simple cases of the Casimir force between 2 scatterers on a 1-dimensional line and between two plane and parallel mirrors coupled through specular scattering to 3-dimensional electromagnetic fields [53].

The first case corresponds to the quantum theory of a scalar field with two counterpropagating components. A mirror is thus described by a  $2 \times 2$   $S$ -matrix containing the reflection and transmission amplitudes  $r$  and  $t$ . Two mirrors form a Fabry-Perot cavity described by a global  $S$ -matrix which can be evaluated from the elementary matrices  $S_1$  and  $S_2$  associated with the two mirrors. All  $S$ -matrices are unitary and their determinants are shown to obey the simple relation

$$\ln \det S = \ln \det S_1 + \ln \det S_2 + i\Delta \quad (3)$$

$$\Delta = i \ln \frac{d}{d^*} \quad , \quad d(\omega) = 1 - r_1 r_2 \exp \left( \frac{2i\omega L}{c} \right)$$

The phaseshift  $\Delta$  associated with the cavity is expressed in terms of the denominator  $d$  describing the resonance effect. The sum of all these phaseshifts over the field modes leads to the following expression of the Casimir free energy  $\mathcal{F}$

$$\mathcal{F} = -\hbar \int \frac{d\omega}{2\pi} N(\omega) \Delta(\omega) \quad (4)$$

$$N(\omega) = \frac{1}{\exp \left( \frac{\hbar\omega}{k_B T} \right) - 1} + \frac{1}{2}$$

Here  $N$  is the mean number of thermal photons per mode, given by the Planck law, augmented by the term  $\frac{1}{2}$  which represents the contribution of vacuum [53].

This phaseshift formula can be given alternative interpretations. In particular, the Casimir force

$$F = \frac{\partial \mathcal{F}(L, T)}{\partial L} \quad (5)$$

can be seen as resulting from the difference of radiation pressures exerted onto the inner and outer sides of the mirrors by the field fluctuations [53]. Using the analytic properties of the scattering amplitudes, the free energy may be written as the following expression after a Wick rotation ( $\omega = i\xi$  are imaginary frequencies)

$$\mathcal{F} = \hbar \int \frac{d\xi}{2\pi} \cot\left(\frac{\hbar\xi}{2k_B T}\right) \ln d(i\xi) \quad (6)$$

Using the pole decomposition of the cotangent function and the analytic properties of  $\ln d$ , this can finally be expressed as the Matsubara sum ( $\sum'_m$  is the sum over positive integers  $m$  with  $m = 0$  counted with a weight  $\frac{1}{2}$ )

$$\mathcal{F} = k_B T \sum'_m \ln d(i\xi_m) \quad , \quad \xi_m \equiv \frac{2\pi m k_B T}{\hbar} \quad (7)$$

The same lines of reasoning can be followed when studying the geometry of two plane and parallel mirrors aligned along the axis  $x$  and  $y$ . Due to the symmetry of this configuration, the frequency  $\omega$ , transverse vector  $\mathbf{k} \equiv (k_x, k_y)$  and polarization  $p = \text{TE, TM}$  are preserved by all scattering processes. The two mirrors are described by reflection and transmission amplitudes which depend on frequency, incidence angle and polarization  $p$ . We assume thermal equilibrium for the whole “cavity + fields” system, and calculate as in the simpler case of a 1-dimensional space. Care has however to be taken to account for the contribution of evanescent waves besides that of ordinary modes freely propagating outside and inside the cavity [7, 54]. The properties of the evanescent waves are described through an analytical continuation of those of ordinary ones, using the well defined analytic behavior of the scattering amplitudes. At the end of this derivation, the free energy has the following form as a Matsubara sum [55]

$$\mathcal{F} = \sum_{\mathbf{k}} \sum_p k_B T \sum'_m \ln d(i\xi_m, \mathbf{k}, p) \quad (8)$$

$$d(i\xi, \mathbf{k}, p) = 1 - r_1(i\xi, \mathbf{k}, p) r_2(i\xi, \mathbf{k}, p) \exp^{-2\kappa L}$$

$$\xi_m \equiv \frac{2\pi m k_B T}{\hbar} \quad , \quad \kappa \equiv \sqrt{\mathbf{k}^2 + \frac{\xi^2}{c^2}}$$

$\sum_{\mathbf{k}} \equiv A \int \frac{d^2 \mathbf{k}}{4\pi^2}$  is the sum over transverse wavevectors with  $A$  the area of the plates,  $\sum_p$  the sum over polarizations and  $\sum'_m$  the Matsubara sum.

This expression reproduces the Casimir ideal formula in the limits of perfect reflection  $r_1 r_2 \rightarrow 1$  and null temperature  $T \rightarrow 0$ . But it is valid and regular at thermal equilibrium at any temperature and for any optical model of mirrors obeying causality and high frequency

transparency properties. It has been demonstrated with an increasing range of validity in [53], [54] and [7]. The expression is valid not only for lossless mirrors but also for lossy ones. In the latter case, it accounts for the additional fluctuations accompanying losses inside the mirrors.

It can thus be used for calculating the Casimir force between arbitrary mirrors, as soon as the reflection amplitudes are specified. These amplitudes are commonly deduced from models of mirrors, the simplest of which is the well known Lifshitz model [34, 35] which corresponds to semi-infinite bulk mirrors characterized by a local dielectric response function  $\varepsilon(\omega)$  and reflection amplitudes deduced from the Fresnel law.

In the most general case, the optical response of the mirrors cannot be described by a local dielectric response function. The expression (8) of the free energy is still valid in this case with some reflection amplitudes to be determined from microscopic models of mirrors. Recent attempts in this direction can be found for example in [56–58].

### The non-specular scattering formula

We now present a more general scattering formula allowing one to calculate the Casimir force between stationary objects with arbitrary non planar shapes. The main generalization with respect to the already discussed cases is that the scattering matrix  $\mathcal{S}$  is now a larger matrix accounting for non-specular reflection and mixing different wavevectors and polarizations while preserving frequency [7, 47]. Of course, the non-specular scattering formula is the generic one while specular reflection can only be an idealization.

As previously, the Casimir free energy can be written as the sum of all the phaseshifts contained in the scattering matrix  $\mathcal{S}$

$$\begin{aligned} \mathcal{F} &= i\hbar \int_0^\infty \frac{d\omega}{2\pi} N(\omega) \ln \det \mathcal{S} \\ &= i\hbar \int_0^\infty \frac{d\omega}{2\pi} N(\omega) \text{Tr} \ln \mathcal{S} \end{aligned} \quad (9)$$

The symbols  $\det$  and  $\text{Tr}$  refer to determinant and trace over the modes of the matrix  $\mathcal{S}$ . As previously, the formula can also be written after a Wick rotation as a Matsubara sum

$$\begin{aligned} \mathcal{F} &= k_B T \sum'_m \text{Tr} \ln \mathcal{D}(i\xi_m) \\ \mathcal{D} &= 1 - \mathcal{R}_1 \exp^{-\kappa L} \mathcal{R}_2 \exp^{-\kappa L} \end{aligned} \quad (10)$$

The matrix  $\mathcal{D}$  is the denominator containing all the resonance properties of the cavity formed by the two objects 1 and 2 here written for imaginary frequencies. It is expressed in terms of the matrices  $\mathcal{R}_1$  and  $\mathcal{R}_2$  which represent reflection on the two objects 1 and 2 and of

propagation factors  $\exp^{-\kappa L}$ . Note that the matrices  $\mathcal{D}$ ,  $\mathcal{R}_1$  and  $\mathcal{R}_2$ , which were diagonal on the basis of plane waves when they described specular scattering, are no longer diagonal in the general case of non specular scattering. The propagation factors remain diagonal in this basis with their diagonal values written as in (8). Clearly the expression (10) does not depend on the choice of a specific basis. Remark also that (10) takes a simpler form at the limit of null temperature (note the change of notation from the free energy  $\mathcal{F}$  to the ordinary energy  $\mathcal{E}$ )

$$F = \frac{d\mathcal{E}}{dL} \quad , \quad \mathcal{E} = \hbar \int_0^\infty \frac{d\xi}{2\pi} \ln \det \mathcal{D}(i\xi) \quad (11)$$

Formula (11) has been used to evaluate the effect of roughness or corrugation of the mirrors [47–49] in a perturbative manner with respect to the roughness or corrugation amplitudes (see the next section). It has clearly a larger domain of applicability, not limited to the perturbative regime, as soon as techniques are available for computing the large matrices involved in its evaluation. It has also been used in the past years by different groups using different notations [39–41, 59, 60]. The relation between these approaches is reviewed for example in [61].

### The lateral Casimir force between corrugated plates

As already stated, the lateral Casimir force between corrugated plates is a topic of particular interest. This configuration is more favorable to theory/experiment comparison than that met when studying the normal Casimir force. It could thus allow for a new test of Quantum ElectroDynamics, through the dependence of the lateral force to the corrugation wavevector [48, 49]. Here, we consider two plane mirrors, M1 and M2, with corrugated surfaces described by uniaxial sinusoidal profiles (see Fig. 1 in [49]). We denote  $h_1$  and  $h_2$  the local heights with respect to mean planes  $z_1 = 0$  and  $z_2 = L$

$$h_1 = a_1 \cos(k_C x) \quad , \quad h_2 = a_2 \cos(k_C(x - b)) \quad (12)$$

$h_1$  and  $h_2$  have null spatial averages and  $L$  is the mean distance between the two surfaces;  $h_1$  and  $h_2$  are both counted as positive when they correspond to separation decreases;  $\lambda_C$  is the corrugation wavelength,  $k_C = 2\pi/\lambda_C$  the corresponding wave vector, and  $b$  the spatial mismatch between the corrugation crests.

At lowest order in the corrugation amplitudes, when  $a_1, a_2 \ll \lambda_C, \lambda_P, L$ , the Casimir energy may be obtained by expanding up to second order the general formula (11). The part of the Casimir energy able to produce a lateral force is thus found to be

$$F^{\text{lat}} = -\frac{\partial \delta \mathcal{E}}{\partial b} \quad (13)$$

$$\delta \mathcal{E} = -\hbar \int_0^\infty \frac{d\xi}{2\pi} \text{Tr} \left( \delta \mathcal{R}_1 \frac{\exp^{-\kappa L}}{\mathcal{D}_0} \delta \mathcal{R}_2 \frac{\exp^{-\kappa L}}{\mathcal{D}_0} \right)$$

$\delta \mathcal{R}_1$  and  $\delta \mathcal{R}_2$  are the first-order variation of the reflection matrices  $\mathcal{R}_1$  and  $\mathcal{R}_2$  induced by the corrugations;  $\mathcal{D}_0$  is the matrix  $\mathcal{D}$  evaluated at zeroth order in the corrugation; it is diagonal on the basis of plane waves and commutes with  $\mathcal{K}$ .

Explicit calculations of (14) have been done for the simplest case of experimental interest, with two corrugated metallic plates described by the plasma dielectric function. These calculations have led to the following expression of the lateral energy

$$\delta E = \frac{A}{2} G_C(k_C) a_1 a_2 \cos(k_C b) \quad (14)$$

with the function  $G_C(k_C)$  given in [49]. It has also been shown that the PFA was recovered for long corrugation wavelengths, when  $G_C(k_C)$  is replaced by  $G_C(0)$  in (14). This important argument can be considered as a properly formulated “Proximity Force Theorem” [49]. It has to be distinguished from the approximation (PFA) which consists in an identification of  $G_C(k_C)$  with its limit  $G_C(0)$ . For arbitrary corrugation wavevectors, the deviation from the PFA is described by the ratio

$$\rho_C(k_C) = \frac{G_C(k_C)}{G_C(0)} \quad (15)$$

The variation of this ratio  $\rho_C$  with the parameters of interest has been described in a detailed manner in [48, 49]. Curves are drawn as examples in the Fig. 1 of [48] with  $\lambda_P = 137\text{nm}$  chosen to fit the case of gold covered plates. An important feature is that  $\rho_C$  is smaller than unity as soon as  $k_C$  significantly deviates from 0. For large values of  $k_C$ , it even decays exponentially to zero, leading to an extreme deviation from the PFA.

Other situations of interest have also been studied. When the corrugation plates are rotated with respect to each other, a torque appears to be induced by vacuum fluctuations, tending to align the corrugation directions [62]. In contrast with the similar torque appearing between misaligned birefringent plates [63], the torque is here coupled to the lateral force. The advantage of the configuration with corrugated plates is that the torque has a larger magnitude. Another case of interest may be designed by using the possibilities offered by cold atoms techniques. Non trivial effects of geometry should be visible in particular when using a Bose-Einstein condensate as a local probe of vacuum above a nano-grooved plate [64, 65].

These results suggested that non trivial effects of geometry, *i.e.* effects beyond the PFA, could be observed with dedicated lateral force experiments. It was however difficult to achieve this goal with corrugation amplitudes  $a_1, a_2$  meeting the conditions of validity of the perturbative expansion. As already stated, recent experiments have been able to probe the beyond-PFA regime with deep corrugations [50, 51] and it also became possible to calculate the forces between real mirrors without the perturbative assumption. In particular, an exact expres-

sion has been obtained for the force between two nanostructured surfaces made of real materials with arbitrary corrugation depth, corrugation width and distance [52].

### The plane-sphere geometry beyond PFA

In the plane-sphere geometry, it is also possible to use the general scattering formula (11) to obtain explicit evaluations of the Casimir force. The reflection matrices may here be written in terms of Fresnel amplitudes on the plane mirror and of Mie amplitudes on the spherical one. The scattering formula is then obtained by writing also transformation formulas from the plane waves basis to the spherical waves basis and conversely. The result takes the form of a multipolar expansion with spherical waves labeled by quantum numbers  $\ell$  and  $m$  ( $|m| \leq \ell$ ). For doing the numerics, the expansion is truncated at some maximum value  $\ell_{\max}$ , which restricts accurate evaluations to a domain  $x \equiv L/R > x_{\min}$  with  $x_{\min}$  proportional to  $1/\ell_{\max}$ .

Such calculations have first been performed for perfectly reflecting mirrors [59, 66]. It was thus found that the Casimir energy was smaller than expected from the PFA and, furthermore, than the result for electromagnetic fields was departing from PFA more rapidly than was expected from previously existing scalar calculations [40, 41]. It is only very recently that the same calculations have been done for the more realistic case of metallic mirrors described by a plasma model dielectric function [43]. Results of these evaluations are expressed in terms of reduction factors defined for the force  $F$  or force gradient  $G$  with respect to the PFA expectations  $F^{\text{PFA}}$  and  $G^{\text{PFA}}$  respectively

$$\rho_F = \frac{F}{F^{\text{PFA}}} \quad , \quad \rho_G = \frac{G}{G^{\text{PFA}}} \quad (16)$$

Examples of curves for  $\rho_F$  and  $\rho_G$  are shown on Fig.2 of [43] for perfect and plasma mirrors.

Using these results, it is possible to compare the theoretical evaluations to the experimental study of PFA in the plane-sphere geometry [44]. In this experiment, the force gradient is measured for various radii of the sphere and the results are used to obtain a constraint  $|\beta_G| < 0.4$  on the slope at origin  $\beta_G$  of the function

$$\rho_G(x) = 1 + \beta_G x + O(x^2) \quad (17)$$

Now the comparison of this experimental information to the slope obtained by interpolating at low values of  $x$  the

theoretical evaluations of  $\rho_G$  reveals a striking difference between the cases of perfect and plasma mirrors. The slope  $\beta_G^{\text{perf}}$  obtained for perfect mirrors is larger than that  $\beta_G^{\text{Gold}}$  obtained for gold mirrors by a factor larger than 2

$$\beta_G^{\text{perf}} \sim -0.48 \quad , \quad \beta_G^{\text{Gold}} \sim -0.21 \quad (18)$$

Meanwhile,  $\beta_G^{\text{Gold}}$  is compatible with the experimental bound obtained in [44] (see [43]) whereas  $\beta_G^{\text{perf}}$  lies outside this bound (see also [59]).

The lesson to be learned from these results is that more work is needed to reach a reliable comparison of experiment and theory on the Casimir effect. Experiments are performed with large spheres for which the parameter  $L/R$  is smaller than 0.01, and efforts are devoted to calculations pushed towards this regime [67].

Meanwhile, the effect of temperature should also be correlated with the plane-sphere geometry. The first calculations accounting simultaneously for plane-sphere geometry, temperature and dissipation have been published very recently [68] and they show several striking features. The factor of 2 between the long distance forces in Drude and plasma models is reduced to a factor below 3/2 in the plane-sphere geometry. Then, PFA underestimates the Casimir force within the Drude model at short distances, while it overestimates it at all distances for the perfect reflector and plasma model. If the latter feature were conserved for the experimental parameter region  $R/L (> 10^2)$ , the actual values of the Casimir force calculated within plasma and Drude model could turn out to be closer than what PFA suggests, which would diminish the discrepancy between experimental results and predictions of the thermal Casimir force using the Drude model.

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